Accuracy of Noise Figure Measurement Systems

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April 1988

RF & Microwave Measurement Symposium and Exhibition

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Abstract: This paper describes methods of evaluating the accuracy of noise figure measurement systems for the microwave frequency range. It also discusses a transistor noise figure test, and SSB and DSB testing. Included are actual plots of data taken from typical test systems. In addition, comments are presented for use of programmable impedance equipment which permits generation of noise figure circles for amplifier design work.

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WHY MEASURE NOISE?

Customer Quote:
"Noise Figure is a MONEY FIGURE
What You Can Charge Is Set By What You Can Measure!"

1.0 Even more important than the customer quote is the effect of noise on the receiving system. Modern receiving systems must amplify, detect and process signals that are often equivalent to levels just a few dB above the power that a 50 ohm resistor would generate when sitting in a cryogenic jar of liquid nitrogen.

The ability to accurately measure the receiving system’s noise figure becomes of utmost importance in these cases. The uniformity of results between different systems such as in R and D, production, QA and the customer is a direct benefit of the attention to accuracy.

2.0 Since S and N are signal and noise powers, one might imagine that they are measured with a type of power meter and require a known power input to establish some reference for signal input power. It also happens that the measurements are susceptible to errors similar to regular power measurements.

2.1 In the figure:

K = Boltzman’s constant
B = Bandwidth
T = Temperature in Kelvins
3.0 This figure shows a typical noise figure measurement system for measuring noise figure from 10 to 18000 Mhz. The noise figure meter is set up to control the LO and the 8971 Noise figure Test Set. The noise source switches from off or 'cold' (equivalent to a room temperature resistor at about 296 K) to 'hot' (equivalent to 9893 K) for a 15.2 dB ENR 346B, where ENR is the excess noise ratio or additional noise power KtB watts over what a resistor would generate at the reference temperature 290.0 K.

One could imagine that a noisy receiver wouldn’t show much difference between the noise source on N2 or off N1, while a quiet receiver would show a large difference between N1 and N2 because it would ‘mask’ the cold source’s power less. The ratio of the two powers N2/N1 is called Y factor and would be the same as the noise source’s ENR for a perfectly quiet receiver.
4.0 There is more to accurate noise figure measurements than just a tight specification for 'instrument uncertainty'. There are other critical parameters which must be considered when overall measurement accuracy is determined. All the factors listed should be taken into account when the accuracy of the noise figure system is being analyzed.

By analyzing the measurement process one can obtain a sense for how consistent the results should be. This allows other problems such as operator error to be identified.

The analysis will indicate how the system can be improved with isolators, preamplifiers or bandpass filtering.
5.0 In order to correct for the measurement system's own noise figure a calibration must be performed before starting the measurement.

5.1 As the figure shows the hot and cold powers N2' and N1' read by the NFM are functions of Te of the system and the following which includes their errors:

5.1.1 Hot noise as seen by the system front end. Note that only the excess hot noise over Tc (or room temperature) gets affected by the input mismatch. This assumes that the noise source doesn't change its match between hot and cold. Note also that Th is the absolute value without calibration error.

\[
Y_2 = \frac{N'2}{N'1} = \frac{KG_2 \cdot B[Thi + Te] \cdot Lin}{KG_2 \cdot B[Tc + Te]} \\
\]

Where: \(Thi = \) Hot Source, Seen By System
\(MMu = (1 + r \cos \theta)^2\)
\(Lin = \) System Linearity

5.1.2 The mismatch uncertainty will be treated as two reflections, the N.S. and the system input, with \(\theta\) being the difference of their phases.

5.1.3 The linearity is a combination of 8970B detector and 8971B mixer linearity.

5.2 Note that KG2B drops out.

6.0 The measurement of the DUT is similar to the calibration except for the new mismatch term corresponding to the output match of the DUT and the input match of the system.
7.0 Finally we are ready to measure some noise figures. The four powers \( N_1, N_2, N_1', \) and \( N_2' \) are used to compute \( F_1, F_2, F_{12} \) and \( G_1. \) \( F_{12} \) is the ‘uncorrected’ noise figure or the total noise figure of the DUT and system. If the DUT has lots of gain \( (G_1) \) than it will mask the effective system temperature \( (T_{sys}) \). If \( G_1*T_{sys} \) is low and \( F_2 \) is high then not accurately correcting for system noise \( F_2 \) leads to large errors in measuring \( F_1. \)

7.1 In the equations ENR_{mea} is the value of ENR stamped on the noise source including it’s measurement error which is typed into the 8970B.

7.2 IFacc is the uncertainty of the IF attenuators in the 8970B. The attenuator values are measured during instrument calibration and stored in ROM. The uncertainty is \( \pm/- 0.15 \text{ dB} \) including environmental and measurement accuracy.

7.3 Grep is the gain repeatability of the system and for the 8970B is typically \( 0.1 \text{ dB} \) for 3 sigma, while the 8971B gain repeatability is about \( 0.2 \text{ dB} \) for 3 sigma in gain for SSB measurements. The errors for the 8971B assume that the frequencies are approached in the same direction, and are the same as when the 8971B was calibrated to avoid hysteresis.

7.4 The formula for \( F_1 \) using the cascade noise equation has many uses such as computing effects of the second stage on overall receiver noise and as we shall see can be used as an accuracy modeling tool.
8.0 These plots show noise figure accuracy NF1 vs the sum of the gain (G1) and noise figure (NF1) in dB of the DUT for several system noise figures (NF23). The errors are plotted for a DUT SWR of 2:1 with a DUT output and system input SWR of 2:1 also. If the SWR in of the DUT is 1:1 the accuracy is about 0.22 dB better and if the DUT SWR is 3:1 the NF1 accuracy is about 0.15 dB worse. (Additional charts are at the back of this paper.)

8.1 The plots were generated using the models in the previous figures with the NF1 accuracy plotted for the 95 percentile of 250 iterations for each 5dB NF1 + G1 point. The system accuracy for gain, linearity, repeatability, and noise source accuracy were varied with a Gaussian distribution (the mean was set to zero with the 3 sigma point set at the commercial specification), while the reflection angle used an equal probability distribution.

8.2 Note that if the system noise figure NF23 is less than NF1 + G1 then the degradation due to system noise figure is negligible.

8.3 A minor error of the chart is caused by holding the NF1 constant at 3 dB and varying G1, and assuming that NF1 accuracy is independent for any combination of NF1+G1. This causes only a small computational error and can be seen as a small decrease in error of about 0.02 dB for a 0 dB system noise figure.

8.4 The chart assumes that sufficient averaging of the measurement has occurred to make noise jitter negligible. The jitter is reduced by the square root of the number of averages taken.
9.0 This is a form of the cascade noise equation showing the degradation of a preamp's noise figure NF2 by the following measuring system's noise figure.

9.1 Since the system noise figure can be improved by selecting a preamp with a gain and noise figure to override the 8971B or other front end high noise, we can use the NF1 accuracy vs NF1+G1 graph and this chart to analyze or synthesize a noise measurement system.

EXAMPLE:

A preamp is available with a 3 dB noise figure and 15 dB of gain. It will be used to decrease the noise figure of a 20 dB 8971B. What is the new system noise figure?

Preamp NF2+G2=3+15=18 dB
8971B NF3 =20 dB

This gives an NF3 contribution to our 3 dB amplifier of 4 dB, therefore:

System NF23 =7 dB

If we're measuring a 2 dB NF, 8 dB gain 2:1 SWR amplifier our accuracy is about +/-0.50 dB
CALCULATING NOISE FIGURE UNCERTAINTY

From the Cascade Noise Figure Equation:
\[ F_1 = F_{12} - \frac{F_2 - 1}{G_1} \]

Taking The Total Differential of \( F_1 \):
\[ \Delta F_1 [\text{dB}] = \frac{F_{12}}{F_1} \Delta F_{12} [\text{dB}] - \frac{F_2}{F_1 \times G_1} \Delta F_2 [\text{dB}] + \frac{F_2 - 1}{F_1 \times G_1} \Delta G_1 [\text{dB}] + \left( \frac{F_{12}}{F_1} - \frac{F_2}{F_1 \times G_1} \right) \Delta \text{ENR} [\text{dB}] \]

Note 'F' and 'G' are Linear Terms, \('\Delta F' and '\Delta \text{ENR}'\) are in Terms of dB.

10.0 The cascade noise figure equation can be differentiated so as to obtain the total differential equation which is useful for evaluating the effects of individual errors on the approximate noise figure measurement error.

10.1 One can see that any errors in measuring \( NF_2 \) are multiplied by \( F_2/(F_1 G_1) \), therefore if \( F_2=32 \ (-15 \text{dB}) \) and \( F_1 G_1 \) is 12 (\( NF_1=10 \), \( G_1=0.8 \text{dB} \)), then the multiplier for \( NF_2 \) error is 2.7, or for every 0.1 dB error in \( NF_2 \) we get 0.27 dB error for \( NF_1 \).

10.2 Fortunately things aren’t quite that bad because of the interaction of the errors, note that a positive error from say detector non-linearity is partially canceled by the sign differences for delta \( NF_1 \) and \( NF_2 \). This is borne out by the statistical method of analyzing errors but the total differential method is more often used. Usually the errors for the differential method are individually squared then summed with the square root taken, called the RSS method. Since the total differential method of solving for the errors assumes the individual error terms are independent, even though they have the same reflection coefficients and scalar errors, the method loses accuracy at the higher multipliers.

**NOISE FIGURE UNCERTAINTY COMPONENTS**

- \( \Delta NF_1 [\text{dB}] \)
  - 1) Mismatch Uncertainty: Noise Source and DUT Input
  - 2) Detector Accuracy

- \( \Delta NF_2 [\text{dB}] \)
  - 1) Mismatch Uncertainty: Noise Source to Meas System
  - 2) Detector Accuracy

- \( \Delta G_1 [\text{dB}] \)
  - 1) Mismatch Uncertainty: Noise Source to DUT Input DUT Output to Meas System
  - 2) Detector Accuracy
  - Gain Repeatability
  - IF Attenuator Accuracy

- \( \Delta \text{ENR} [\text{dB}] \)
  - 1) Calibration Accuracy
  - Connector Wear Etc.
HIGH REFLECTION MEASUREMENT PROBLEMS

Gain and Noise Figure May Be a Function of the Source Impedance.
The Impedance of Some Noise Sources Changes between $T_c$ and $T_h$.
Especially a Problem With Unmatched Transistors
A) Use the HP 346A Which Changes Reflection Coefficient Less Than 0.01.
B) Install a Well Matched Calibrated Isolator on the Output of Poorly Matched DUTS.
The 8970A/B Can Compensate for Loss if Known.

11.0 When measuring high reflection, low gain devices the slight change in reflection (-0.04) that occurs for a 15.2 dB 346B noise source may cause errors. The cause is the computed gain, $G_1$, will incur an additional mismatch error, causing an error in $N_{Fl}$ of the device. This is a problem with amplifiers which change their gain rapidly for small changes in input match. If the amplifier has a low stability factor this is a clue that the problem may occur. See N. Kuhn; "Curing a Subtle but Significant Cause of Noise Figure Error"; Microwave Journal; June 1984.

11.1 If the isolator route to improving the accuracy is chosen be aware that the bandwidth will be less than the bandwidth of the wideband noise source, and will often rise sharply in loss and drop in isolation out of band. The reflection coefficient may be higher for the isolator than the noise source. The isolator may not have instrumentation grade connectors and may be suspect after a few connections.

12.0 A major discrepancy in measuring noise figure in the past has been the effect of changes in noise figure between the desired IF passband and the undesired image band and other spurious responses, from the mixer and LO. Remember, a wide band noise source will mix with any LO spurious or LO noise.

12.1 This graph shows the effect on noise measurements if the noise is different in the spurious channel than in the desired channel for several gain differences between channels. Note that if the noise is constant, gain differences have no effect.

The graph plots $\Delta N_F$ derived from:

\[ (G_1+G_2)T_h+G_1T_e1+G_2T_e2 \]
\[ (G_1+G_2)T_c+G_1T_e1+G_2T_e2 \]

Where: $G_1$, $T_e1$ Desired channel
$G_2$, $T_e2$ Undesired channel

12.2 Example:
Desired channel $N_F$ 8.0 dB
Spurious channel $N_F$ 10.0 dB
$\Delta N_F = 10 - 8 = 2.0$ dB
Desired channel gain 20.0 dB
Spurious channel gain 15.0 dB
$\Delta G = 15 - 20 = -5.0$ dB
Error $\sim +0.6$ dB
SOLUTIONS TO REDUCING ERRORS

A) Minimize the Noise Figure of the Test System  
   Especially Important for Low Gain + Low Noise DUTS  
B) Reduce the SWR of all Mismatches if Possible  
C) Evaluate the Test System Carefully and Eliminate  
   Any Sharp Resonances  
D) Calibrate the System at Every Frequency  
E) Use Synthesized Sources for Critical Applications

PROTECTING YOUR ACCURACY

A) ENR Calibration Uncertainty  
   Recommend Recalibration After 1000 (APC-3.5) or  
   2000 (APC-7 or N) Connections.  
   Keep 1 to Use as a Standard to  
   Check the Working Units  
B) Connector Wear  
   Train Operators in Good Connector Habits.  
   Cleanliness, Proper Torque,  
   Turn the Nut (Not the Housing).  
C) Avoid the Use of Adapters, or Use an APC-7  
   With Precision Adapters, and Calibrated Loss.

INSERTION GAIN VS AVAILABLE GAIN

A Component of Most High Frequency Transistor  
Amplifier Design Analysis and Synthesis Formulas is  
Available Power Gain of the Active Device Used.  

\[ G_a = \frac{P_{ao}}{P_{as}} = \frac{\text{Power Available From the Output}}{\text{Power Available from the Source}} \]

\[ = \frac{1 - \left| S_{21} \right|^2}{1 - \left| S_{22} \right|^2} \approx \frac{\left| S_{21} \right|^2}{\left| S_{22} \right|^2} \]

Where:  
\[ \Gamma_0 = S_{22} + \frac{S_{12} S_{21}}{1 - \left| S_{11} \right|^2} \]

Unless a Tuner is Installed On the Device Output, the  
Noise Figure Meter Will Measure Insertion Gain  
(Approx) \[ \left| S_{21} \right|^2 \]

13.0 Pay attention to system noise figure to get it below the NFl+Gl flat error knee, then reduce mismatches.  
Don’t depend on interpolation between measured points and if measuring with a YIG tuned filter or oscillators,  
approach measured frequencies in the same direction as they were calibrated.

14.0 In a production environment where the noise sources may see a lot of use (abuse?), it is a good idea to have a standby standard.

14.1 The front face of high contact pressure connectors, SMA or APC 3.5 should not be rotated excessively against the mated connector.

14.2 Most APC 7(R) connector adapters have lower SWR than their low priced N counterparts and probably should be used in critical applications where several connector options are necessary. Be sure to input their loss to the 8970B.

15.0 The NFM measures the insertion gain, while the desired term for low noise amplifier design is usually available gain Ga. Note that the equation shows Gamma source, which is the reflection from an adjustable tuner presented to the terminals of the device, which after the gain is tuned for maximum is then measured on a network analyzer.

15.1 Note that Fmin, the minimum noise is obtained in the same manner as Ga except the tuner is tuned for minimum noise with the tuner at the input.

15.2 In order to get any degree of accuracy a SSB measurement is necessary when making this measurement, or the admittance measurements of the tuner are meaningless.
16.0 In general, the noise figure of a transistor is a function of the source admittance $Y_s = G_s + B_s$ presented to the input. $F_{\text{min}}$ is the minimum noise figure which occurs when $G_s = G_{\text{opt}}$ and $B_s = B_{\text{opt}}$. $R_n$ is the equivalent noise resistance which determines the sensitivity of $F$ to variations in $Y_s$.

16.1 The second equation is derived from the first by expressing the admittances in terms of complex reflection coefficients $\Gamma$'s and $\Gamma_{\text{opt}}$. The parameter $N$ is the normalized equivalent noise resistance, determines the sensitivity of $F$ to variations in $\Gamma$'s.

16.2 Using these expressions, the noise figure for any source admittance can be determined from 4 parameters: $F_{\text{min}}$, $R_n$, $G_{\text{opt}}$ and $B_{\text{opt}}$ (or $F_{\text{min}}$, $N$, $\text{Re}(\Gamma_{\text{opt}})$ and $\text{Im}(\Gamma_{\text{opt}})$). Full noise characterization of a transistor involves the measurement of these 4 parameters, from which a family of constant noise figure circles can be derived. A similar set of circles can be derived for maximum available gain.
17.0 The setup includes a tuner at the input to vary the effective source admittance and one at the output to measure maximum available gain.

17.1 In the first method, the admittance for which the noise figure is a minimum must be set with tuner #1. The tuner admittance is measured on an ANA and \( F_{\text{min}} \) is determined after correcting for the loss of the tuner at that setting. The 4th parameter \( N \) is determined by measuring \( F \) at any other convenient point, such as for a 50 ohm source impedance.

17.2 In the second method, a number of measurements (typically between 7 and 20) are made at pre-calibrated points and a least-squares curve fitting procedure is used to determine the 4 parameters. Maximum available gain parameters can be determined simultaneously with the adjustment of tuner #2 for maximum gain at each point. (Tuner #2 is not required if available gain is not needed.)

17.3 For accuracy, it is important that these measurements be done single sideband (SSB) or the results will be an average over two sidebands separated by \( 2F_{\text{IF}} \). Furthermore, when there is a length of transmission line between the tuner and the transistor, the difference in phase angle of \( s \) between the two sidebands:

\[
\Delta \phi = 4.8^o/\text{Mhz of FIF/meter}
\]
ADVANTAGES OF TRANSISTOR NOISE MEASUREMENT METHOD (B)

- Tuner #1 needs to be set to and calibrated at only n fixed points
- No need for ANA on continuous basis
- Method (A) may be inaccurate if minimum is shallow
- Measurement (B) is easier to automate (though requiring more computation)
- Setting of tuner arrived at in method (A) may not correspond exactly to $\Gamma_{\text{opt}}$ for transistor because of variation of loss of tuner #1

18.0 Compared with method (A), method (B) offers advantages for automation and speed because time and effort does not have to be spent varying tuner #1 to find $\Gamma_{\text{opt}}$. Method (B) requires only that the tuner be set repeatably to each of n calibrated settings. Impedance and loss are previously measured so that the ANA is not required after the initial calibration.

18.1 In addition to the inaccuracy of method (A) when the minimum is broad in the $\Gamma$ plane, the fact that the tuner’s loss varies with setting in some unknown fashion which is not accounted for in finding $F_{\text{min}}$ means that the exact value of $\Gamma_{\text{opt}}$ will not be found: the tuner will be adjusted to a point which minimizes the overall $F$ (which includes the contribution of the tuner).
19.0 The noise source tuner combination can be calibrated for ENR by measuring the NF of the measuring system with a matched source and with the source tuner to be calibrated. Measure the reflection coefficient of the system input and the reflection coefficient of the source tuner combination. The ENR of the source tuner can now be determined. Note that this method requires that the system input be isolated from changes in source match with an isolator.
20.0 The figure shows an automatic microwave transistor characterization test set, NP4 series which automatically draws curves of constant Ga and constant NF across approximately octave bands. The manufacturer is the ATN Company of Lexington Mass. The president is Vahe Adamian. The test set works very well with the 8970B/8971B combination.

20.1 The calibration procedure consists of an initial calibration of a 16 state admittance standard with an 8510. This is probably every 6 months or so. Then a noise source, open circuit, short circuit, and through line is connected. This calibrates the 30 dB ENR source, references the reflectometer for the output match test and sets the gain reference. All the values are stored on a computer as constants for a computer program which is part of the package. The nice touch is the 16 state addmittance and the computer curve fitted approximation to $F_{\text{min}}$ and $G_{\text{amax}}$. The analysis is similar to the constant gain and noise figure measurements described in 15.0 and in G. Gonzalez; Microwave Transistor Amplifiers; Prentice Hall.

21.0 This plot shows constant Ga and NF curves for an NEC transistor at 8 GHz. The $G_{\text{amax}}$ occurs at $.5+j1$, with $F_{\text{min}}$ at about $1+j1$. It is easy to see that an input matching network could be designed that would give good gain and noise figure with little compromise.
22.0 This figure shows a measurement of an attenuator in the 26.5 to 40 GHz frequency range. The source is the new R347B Noise Source. Note the excellent noise figure accuracy. The Mixer was a Honeywell A5100U with a 7 dB system noise figure including the isolator. Since the SWR of the isolator, DUT, and R347B is low, such good performance can be expected.
23.0 These figures show the measurement system and performance of a single sideband noise measurement over 33 to 38 GHz using Q band waveguide and mixer components. The R362A filter rejects the upper sideband. Note this technique requires a high IF frequency: in our case 6450 to 11450 MHz, which means a high IF noise figure. The advantage is that the 1st LO can be at a fixed frequency. The system noise figure is about 16 dB, which really makes the system unreliable for low Gl+NFl type devices (such as the 8 dB attenuator we show it measuring).
ROOM FOR A MIXER MEASUREMENT SYSTEM

24.0 This figure shows a measurement of a mixer using the same system as above, but in this case the system NF was about 7 dB, or the NF of the Avantek LNA. The 1st LO was at 33.06 GHz, with the measurement frequency being 41 to 44 GHz.

24.1 The millimeter measurements were provided courtesy of Peter Tong, who is also the designer of the new Q and R band 346B Noise Sources.

24.2 One thing to watch out for on building your own systems is to avoid the possibility of noise power from the noise source leaking through the mixer into the IF amp; it can cause several dB of uncertainty in your measurements. Most waveguide systems don't suffer from this problem because of the waveguide cutoff frequency. In coax when measuring broadband amplifiers this can be a problem unless there is filtering of the IF, or if the mixer IF to signal isolation times the difference in gain of the DUT at the measurement and IF frequency isn't great. EG, if the Mixer has 20 dB of isolation, but the DUT rolls up in gain 20 dB and its noise changes, errors will be introduced. See 12.1.
NF1 ACCURACY VS NF1+G1[dB]
NF DUT=3; 8971B SYSTEM ACCURACY
DUT OUTPUT SWR=1; PRE AMP SWR=1

DEVICE IS A AMP

SYSTEM CONDITIONS
MEAS SYS UNC dB = .1
ENV DRIFT dB = .17
FREQ SENSING dB = .05
ENR dB = 14.5
DUT N. S. UNC dB = .3
IF N. S. UNC dB = .3
Rho NS = .11
8970 GAIN REPEAT = .1
IF ATTEN UNC dB = .15
PERCENTAL GOOD = 95
DUT IN SWR = 3
DUT IN SWR = 2
DUT IN SWR = 1
DEVICE IS A AMP

SYSTEM CONDITIONS
MEAS SYS UNC dB= .1
ENV DRIFT dB= .17
FREQ SENSIB dB= .05
ENR dB= 14.5
DUT N. S. UNC dB= .3
IF N. S. UNC dB= .3
Rho NS = .11
8970 GAIN REPEAT = .1
IF ATTEN UNC dB= .15
PERCENTAL GOOD = 95
DUT IN SWR = 3
DUT IN SWR = 2
DUT IN SWR = 1
NOISE FORMULAS

\[
NF = 10 \times \log(F) \text{ dB} \\
F = 10^{\frac{NF}{10}} \\
Te = \frac{Th - Y \times Tc}{Y - 1} \\
F = F_{12} - \frac{F_2 - 1}{G_1} \\
F = \frac{Te}{290} + 1 \\
Te = (F - 1) \times 290 \text{ Kelvins} \\
Th = [10^{\frac{ENR}{10}} + 1] \times 290 \text{ Kelvins} \\
F_{enr} = 10^{\frac{ENR}{10}} \\
JITTER (NO AVER) = 3s = (F_{12}/F_{enr} + 1) \times 0.08 \text{ dB} \quad 8970A/B \\
= TYPICAL JITTER FOR 1 AVERAGE IN dB.
\]

NUMBER AVER FOR JITTER e IN dB (96 % APPROX)

\[
N = \left[\frac{1.96 \times s}{e}\right]^2
\]

GENERAL FORM FOR F AT SOME \( r_s \)

\[
F = F_{\text{min}} + \frac{4 \times \frac{r_n}{r_s} \left| r_s - r_o \right|^2}{(1 - \left| r_s \right|^2) \left(1 + \left| r_o \right|^2\right)} \\
\left| r_o \right|^2 = \left| r_s \right|^2 @ F_{\text{min}} \\
r_n = R_n / Z_{\text{ref}}
\]

\[
r_n = (F - F_{\text{min}}) \left| 1 + r_o \right|^2 \\
4 \left| r_o \right|^2
\]

CONSTANT NOISE FIGURE CIRCLES

\[
\text{Cent F}_{\text{min}} = \frac{\left| r_o \right|}{1 - N_i} \\
N_i = \frac{\left| r_s - r_o \right|^2}{1 - \left| r_s \right|^2} \\
\text{Rad F} = \frac{1}{1 + N_i} (N_i^2 + N_i(1 - \left| r_o \right|^2)^2)^2
\]

23